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NeuroSymbolic Artificial Intelligence at Scale

Marco Fanfani, marco.fanfani@unifi.it

https://www.disit.org/

Parte: 2.5 (2025-26) Physics-Informed Neural Networks
Research @ DISIT Lab

Physics-Informed Neural Networks (PINNs)

Solving Partial Differential Equations through Scientific Machine Learning

Results from Botarelli et al. "Using Physics-Informed neural networks for solving Navier-Stokes equations in fluid dynamic complex scenarios"

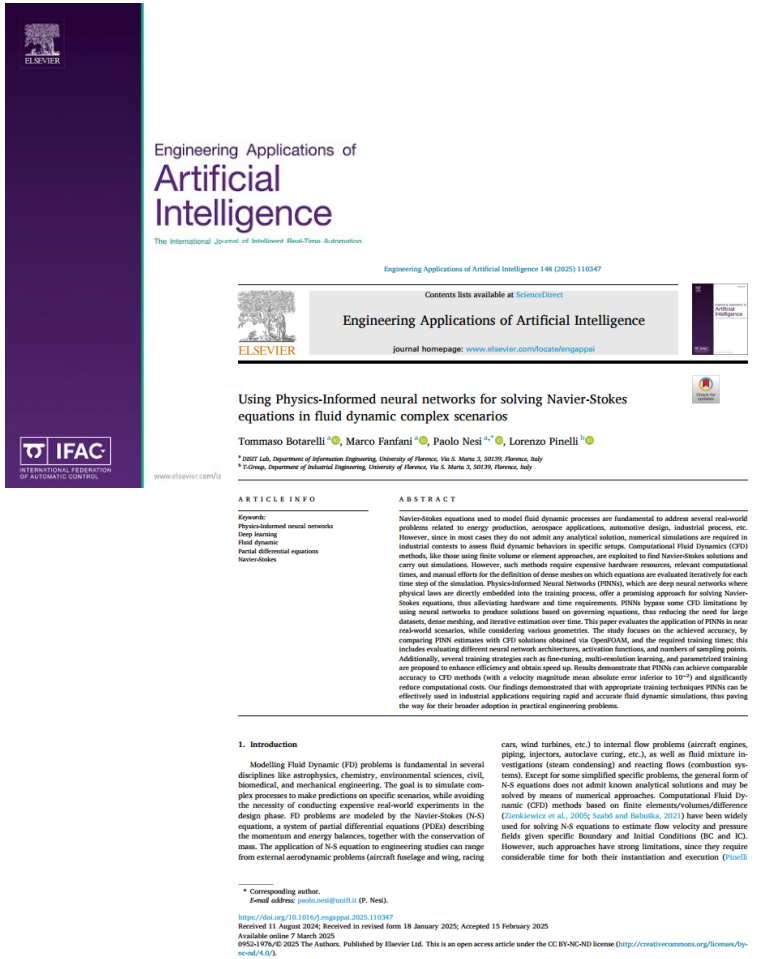
Research @ DISIT Lab

- Research on PINN carried out at the DISIT Lab (Dept. Information Eng.) in collaboration with the T-Group (Dept. Industrial Eng.)

- **Objective**: demonstrating the applicability of PINN on complex fluid-dynamic scenarios

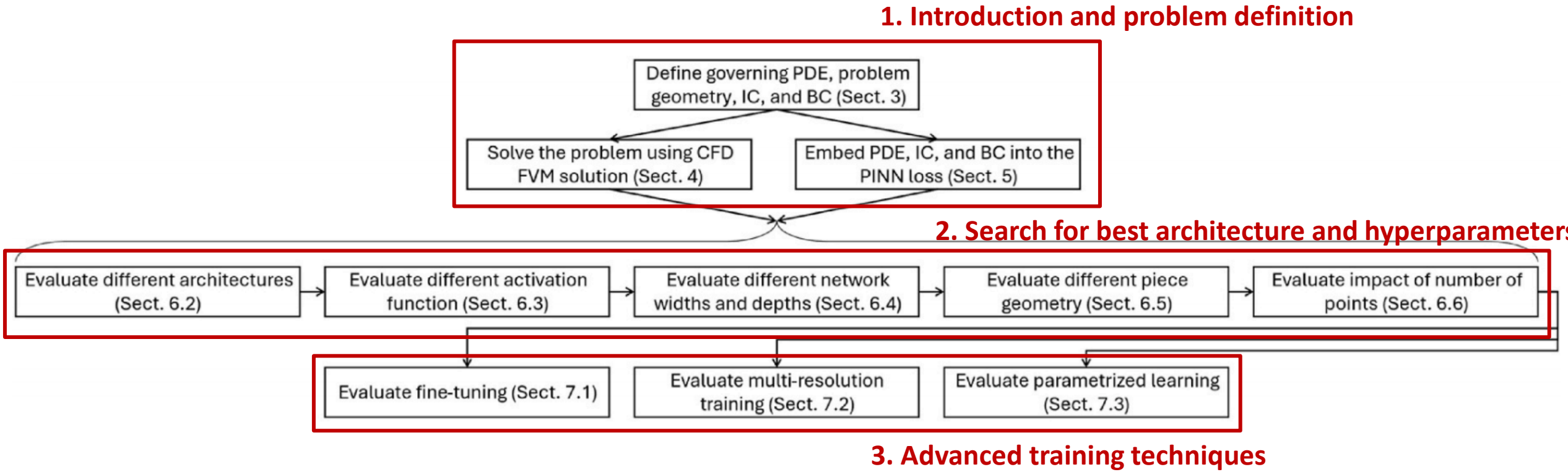
- T. Botarelli, M. Fanfani, P. Nesi, L. Pinelli, "*Using Physics-Informed neural networks for solving Navier-Stokes equations in fluid dynamic complex scenarios*," Engineering Applications of Artificial Intelligence, Volume 148, 2025, 110347, ISSN 0952-1976, https://doi.org/10.1016/j.engappai.2025.110347

- Videos: https://www.snap4city.org/drupal/node/1010



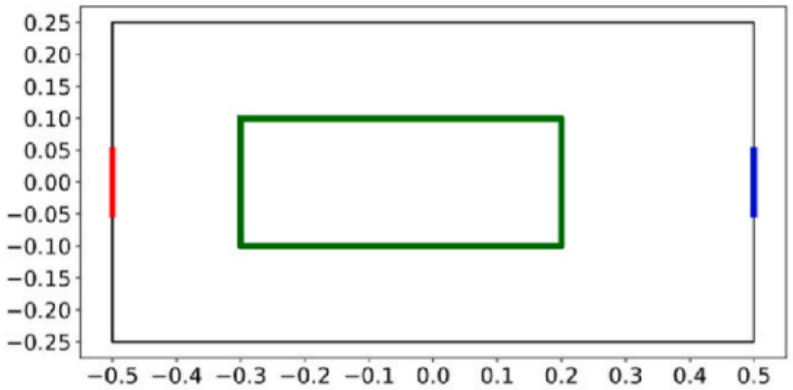
Experiments @ DISIT Lab

- Research project structure

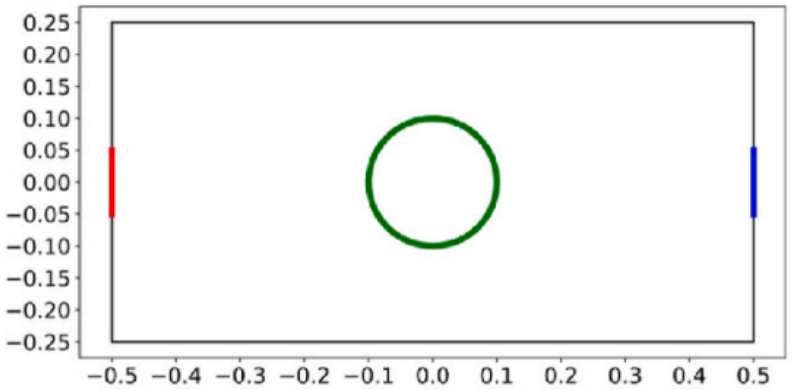


Experiments @ DISIT Lab

- Case studies
  - Solving **Navier-Stokes** estimating u_x, u_y, and p in a forward problem
  - Considering **three different domain** setup
  - Exploiting **ReLoBraLo** and **multiple time windows**



(a)



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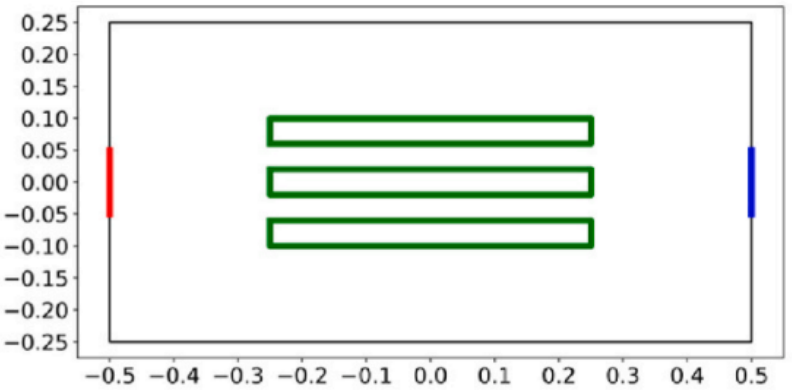
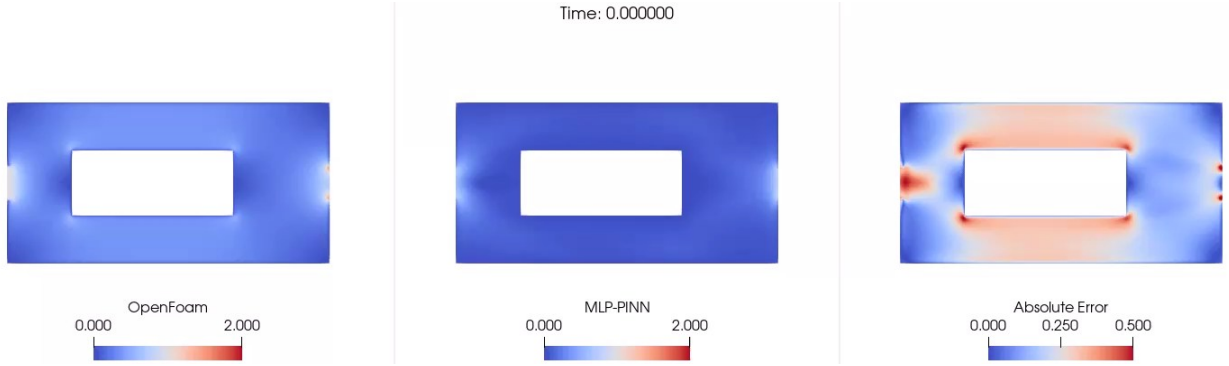
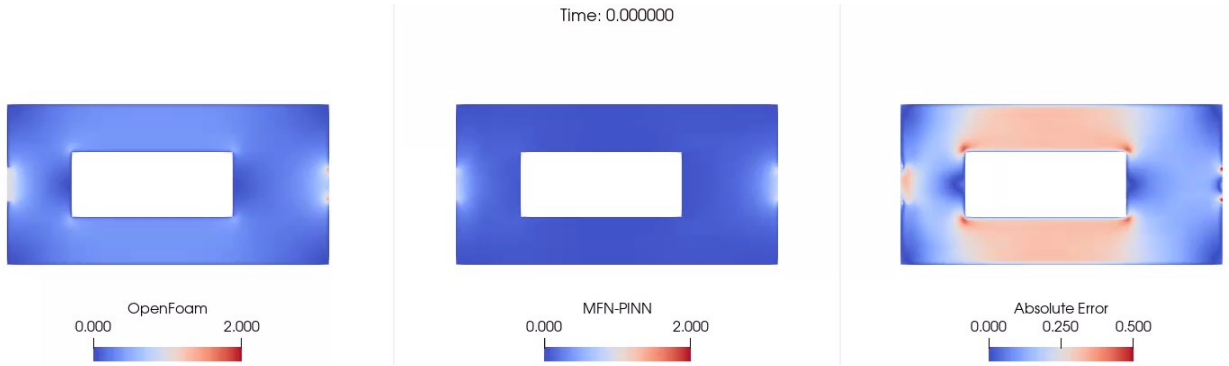


(c)

Network Architecture

- Multi Layer Perception vs Modified Fourier Network

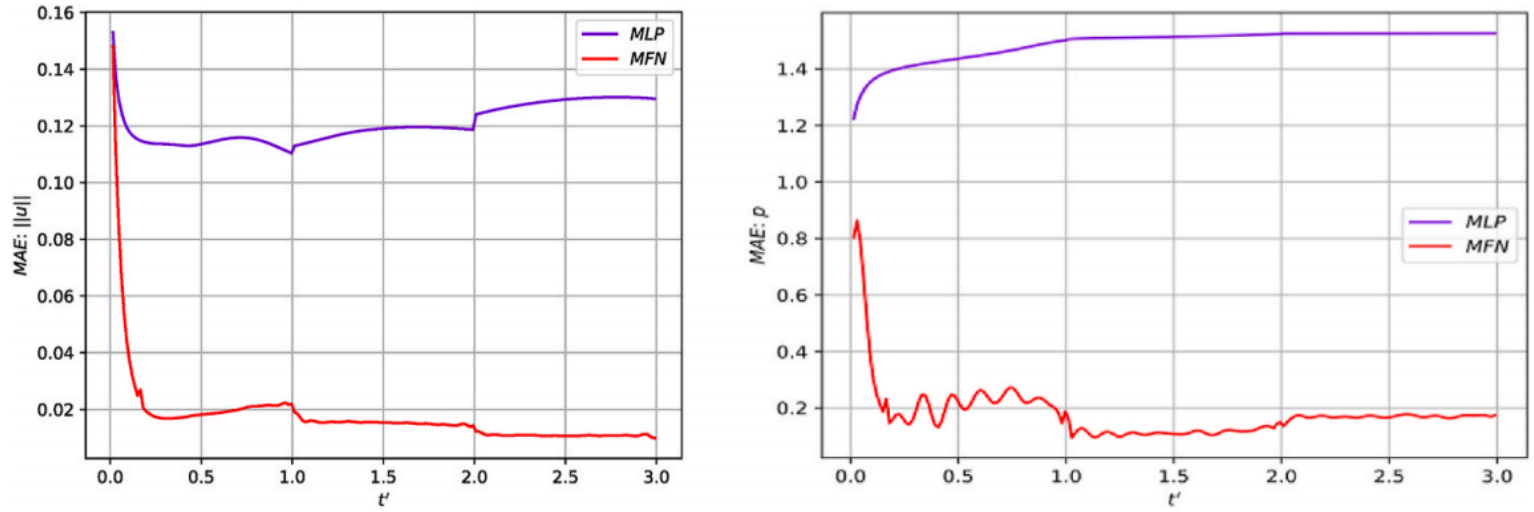




- 6 layers with 512 neurons
- Tanh activation

Network Architecture

- Multi Layer Perception vs Modified Fourier Network

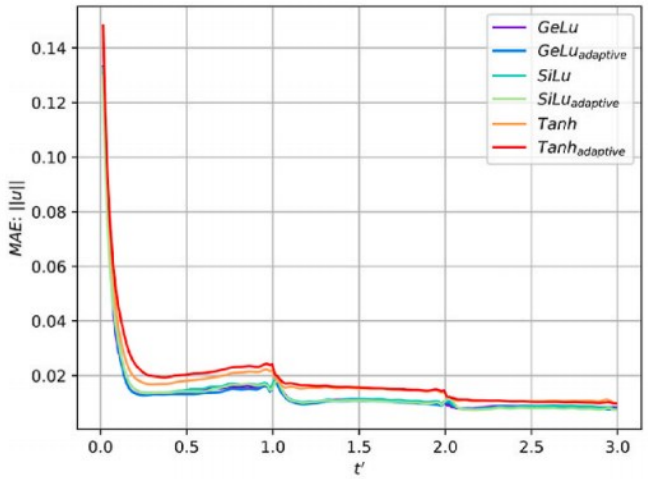


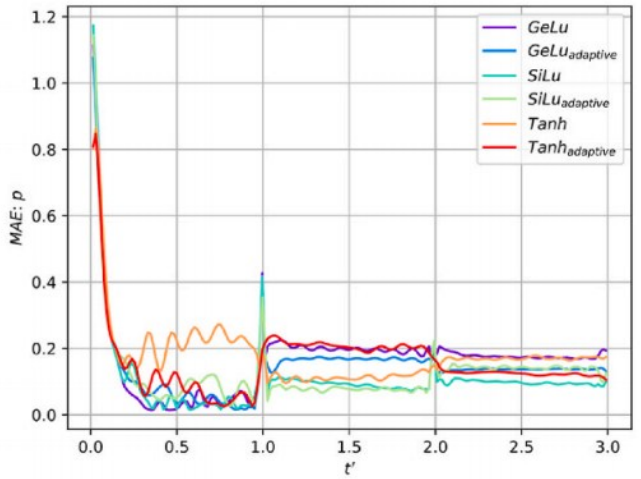
Architecture	MAE_{avg}				MAE_{st}				Training Times (hh:mm)
	$\|\mathbf{u}\|$	u_x	u_y	p	$\|\mathbf{u}\|$	u_x	u_y	p	
MLP-PINN	0.121	0.111	0.053	1.489	0.130	0.119	0.054	1.526	**03:32**
MFN-PINN	**0.017**	**0.017**	**0.009**	**0.179**	**0.011**	**0.010**	**0.005**	**0.171**	04:55

Network Architecture

- Activation Functions

Activations	MAE_{avg}				MAE_{st}				Training Times (hh:mm)
	$\|\|\mathbf{u}\|\|$	u_x	u_y	p	$\|\|\mathbf{u}\|\|$	u_x	u_y	p	
Tanh	0.017	0.017	0.009	0.179	0.011	0.010	0.005	0.171	**04:55**
Tanh Adaptive	0.018	0.017	0.011	0.158	0.010	0.010	0.005	0.119	05:32
SILU	0.014	0.013	**0.007**	**0.107**	0.009	0.008	0.005	**0.093**	05:47
SILU Adaptive	**0.013**	0.013	**0.007**	0.126	**0.008**	**0.007**	**0.004**	0.138	06:10
GELU	**0.013**	0.013	**0.007**	0.166	0.009	0.008	0.005	0.174	05:26
GELU Adaptive	**0.013**	**0.012**	**0.007**	0.145	**0.008**	0.008	**0.004**	0.138	05:49

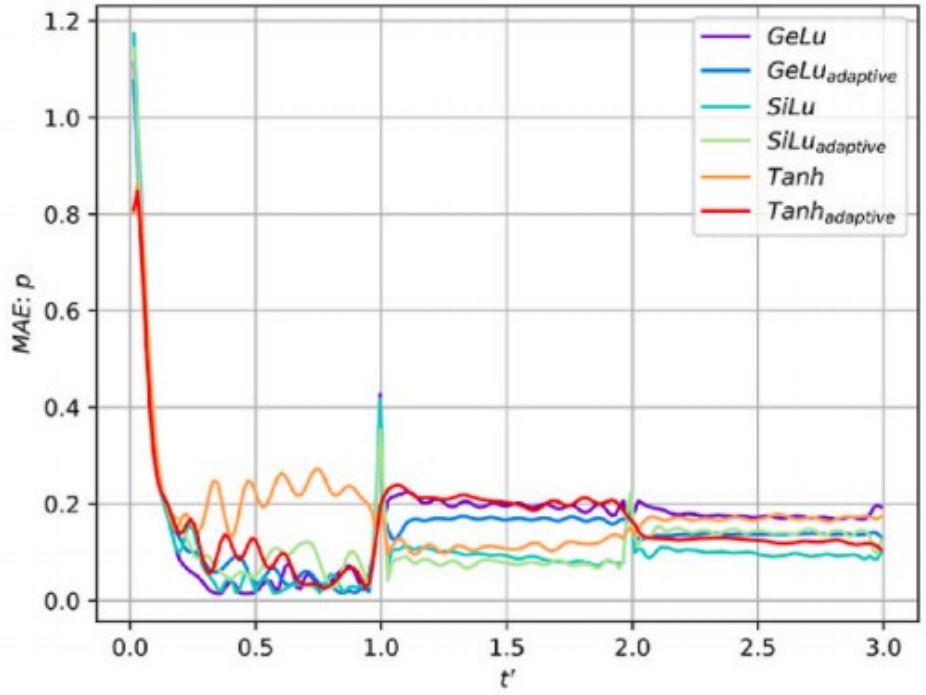




- MFN
- 6 layers with 512 neurons
- Varying activations

Observation on Pressure

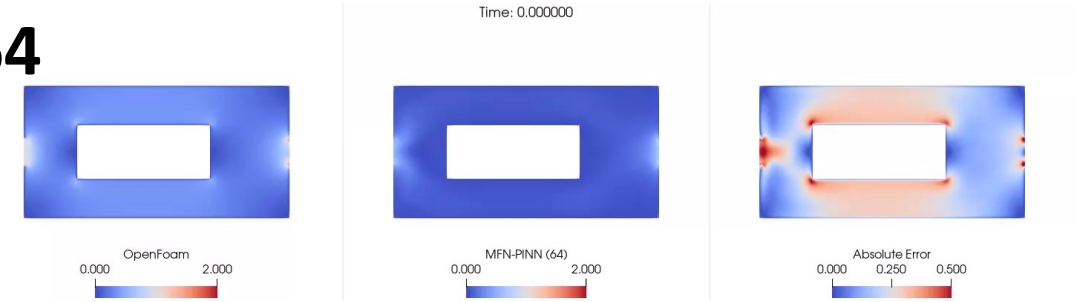
- Pressure estimate always show **more erratic results**
- Estimating pressure is more challenging for PINNs primarily because **no specific pressure constraints are typically defined** in the Initial Conditions (IC) or Boundary Conditions (BC)
- The system is left to manifest pressure conditions solely **as a byproduct of the fluid flow solution**
- Additionally, the **dynamic range of pressure values in these scenarios is significantly wider** (e.g., between -8.816 and 1.390) compared to velocity magnitudes (0.000 to 1.996), which leads to less regular trends and higher relative errors during training



Network Architecture

- Network layer width

64



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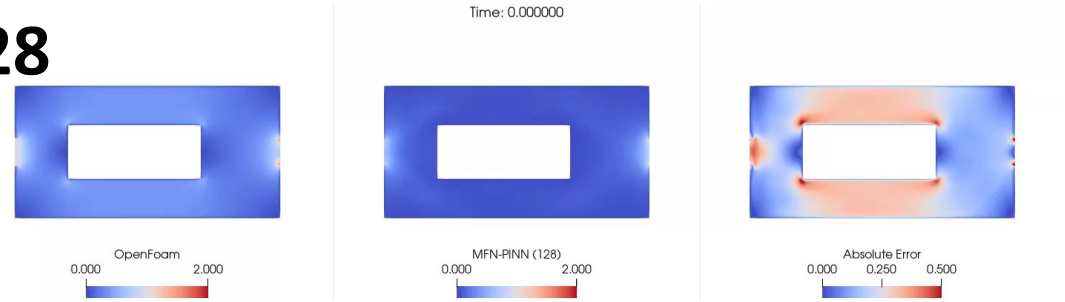

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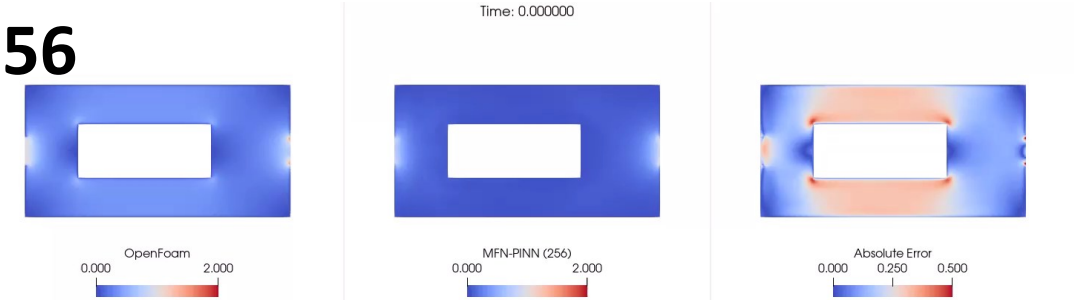

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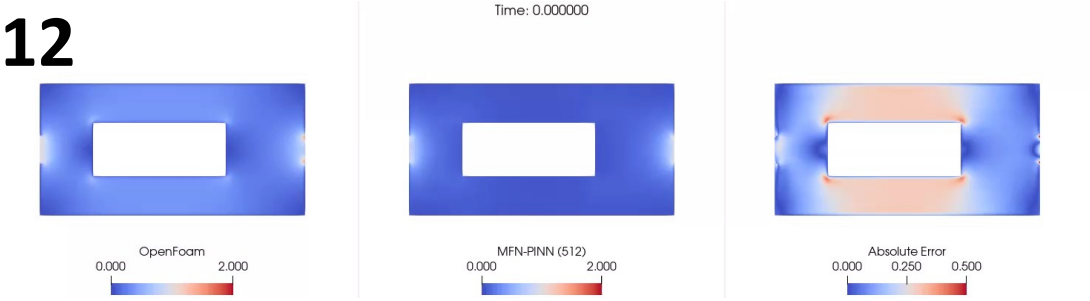
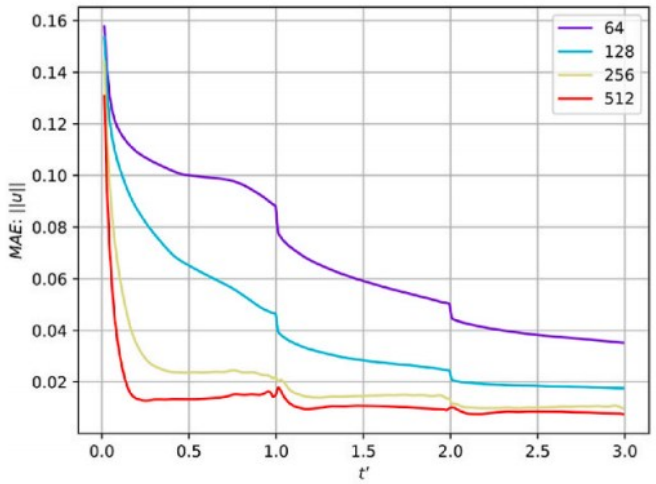


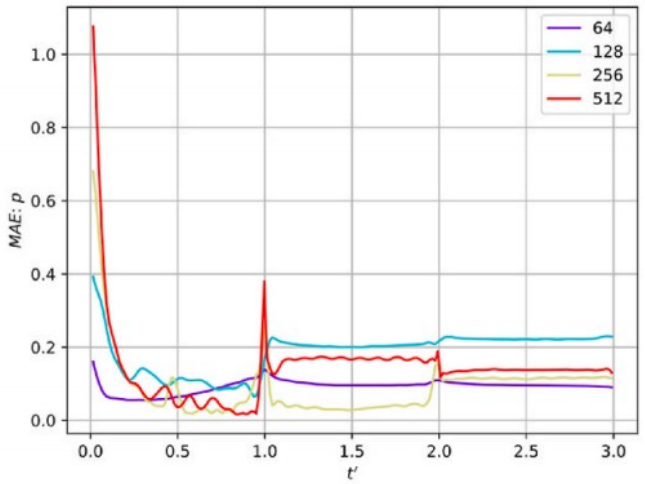
- MFN
- 6 layers with different width
- Adaptive GELU

Network Architecture

- Network layer width

Layer width	MAE_{avg}				MAE_{st}				Training Times (hh:mm)
	$\|\|\mathbf{u}\|\|$	u_x	u_y	p	$\|\|\mathbf{u}\|\|$	u_x	u_y	p	
64	0.068	0.064	0.032	0.092	0.037	0.034	0.018	**0.094**	**03:24**
128	0.040	0.038	0.020	0.187	0.018	0.017	0.009	0.223	03:38
256	0.019	0.019	0.010	**0.091**	0.010	0.010	0.005	0.116	04:25
512	**0.013**	**0.012**	**0.007**	0.145	**0.008**	**0.008**	**0.004**	0.138	05:49



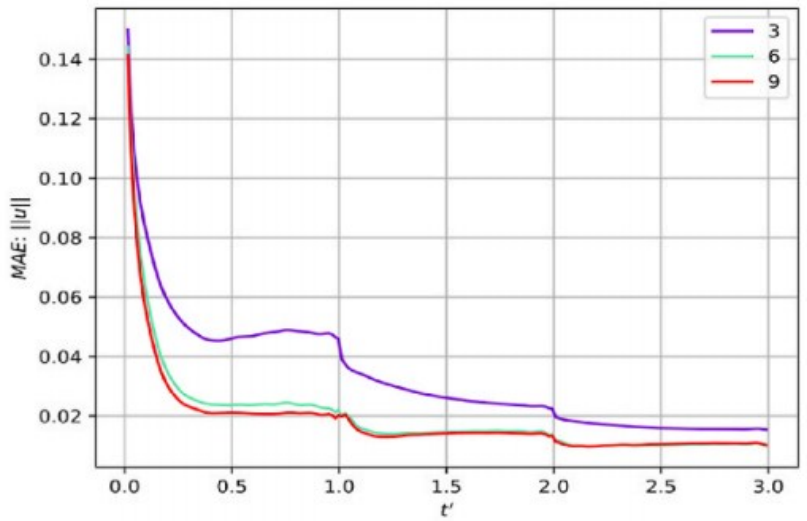


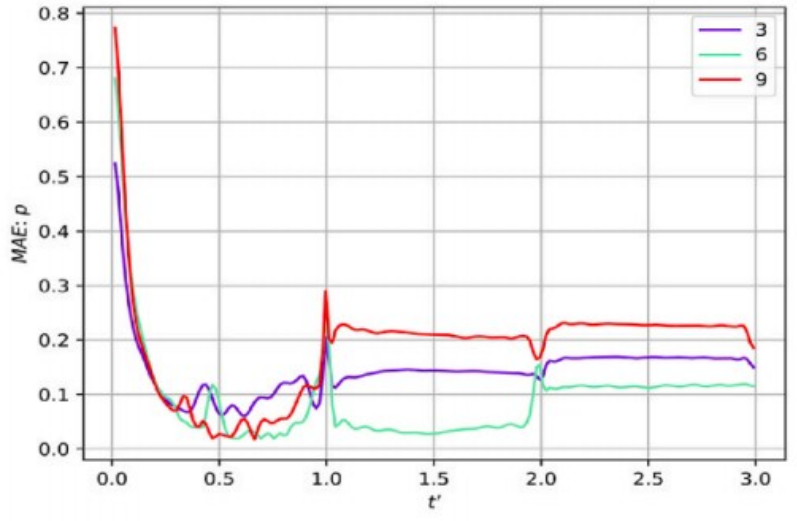
- MFN
- 6 layers with different width
- Adaptive GELU

Network Architecture

- Network depth (layer number)

Hidden layer number	MAE_{avg}				MAE_{st}				Training Times (hh:mm)
	$\|\|\mathbf{u}\|\|$	u_x	u_y	p	$\|\|\mathbf{u}\|\|$	u_x	u_y	p	
3	0.033	0.031	0.017	0.144	0.016	0.015	0.007	0.166	**03:22**
6	0.019	0.019	**0.010**	**0.091**	**0.010**	**0.010**	**0.005**	**0.116**	04:25
9	**0.018**	**0.017**	**0.010**	0.185	**0.010**	**0.010**	**0.005**	0.234	05:18

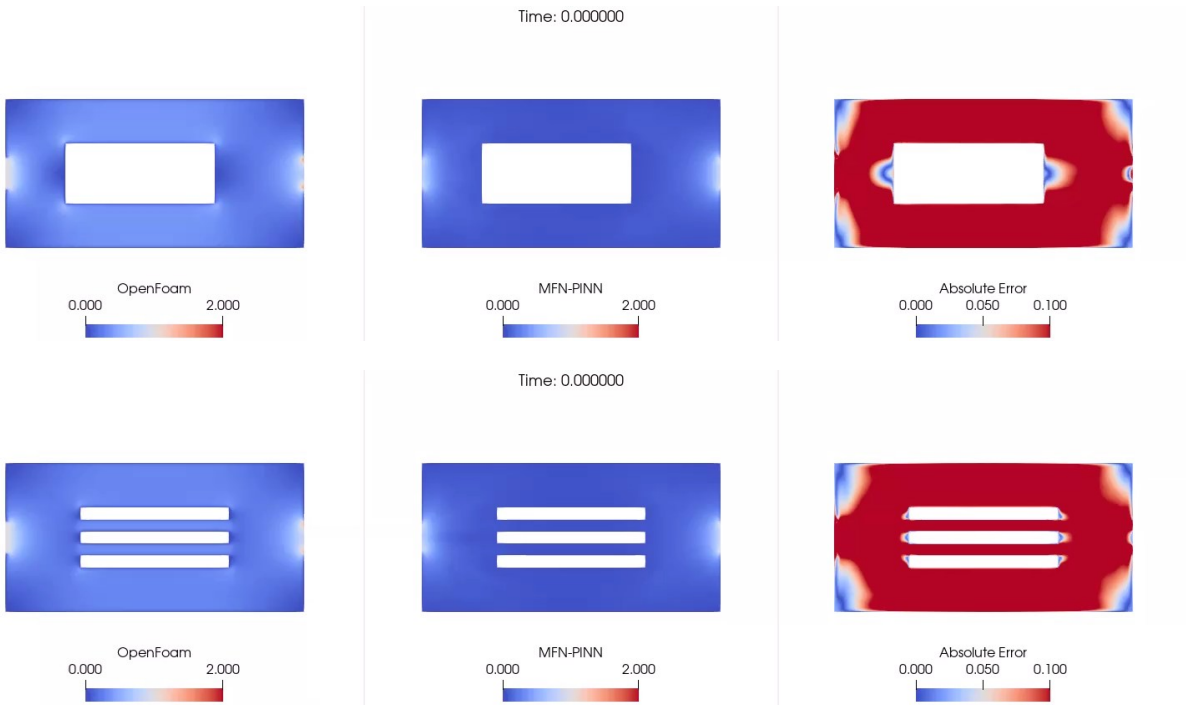




- MFN
- Different number of layers with 256 width
- Adaptive GELU

Domains

- Evaluation on different domains

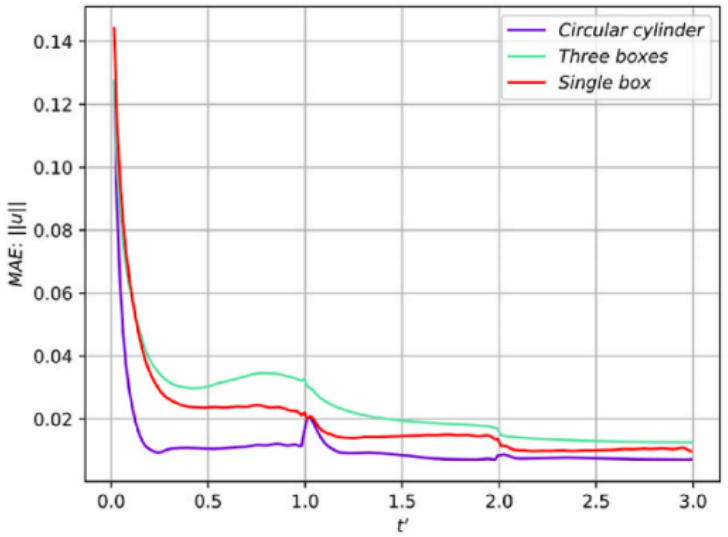


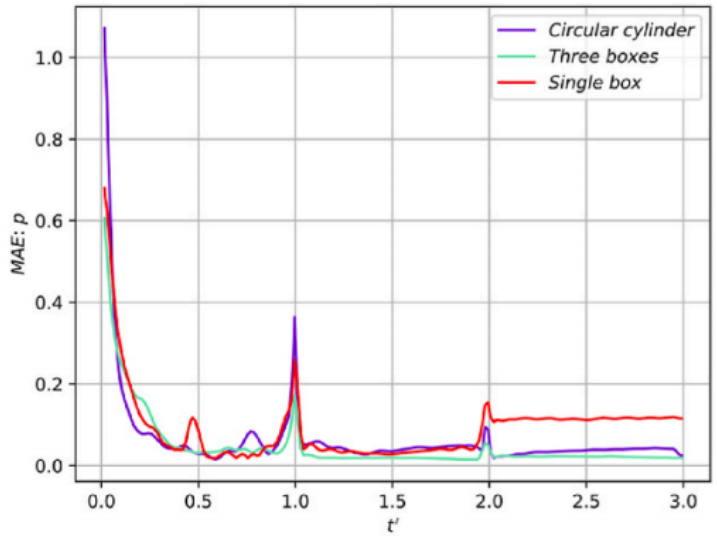
- MFN
- 6 layers with 256 width
- Adaptive GELU
- Evaluated on
  - single box
  - cylinder
  - three-boxes

Domains

- Evaluation on different domains

Piece	MAE_{avg}				MAE_{st}			
	$\|\|\mathbf{u}\|\|$	u_x	u_y	p	$\|\|\mathbf{u}\|\|$	u_x	u_y	p
Single box	0.019	0.019	0.010	0.091	0.010	0.010	0.005	0.116
Circular cylinder	0.011	0.010	0.006	0.065	0.007	0.006	0.004	0.040
Three boxes	0.024	0.023	0.014	0.048	0.013	0.012	0.006	0.021

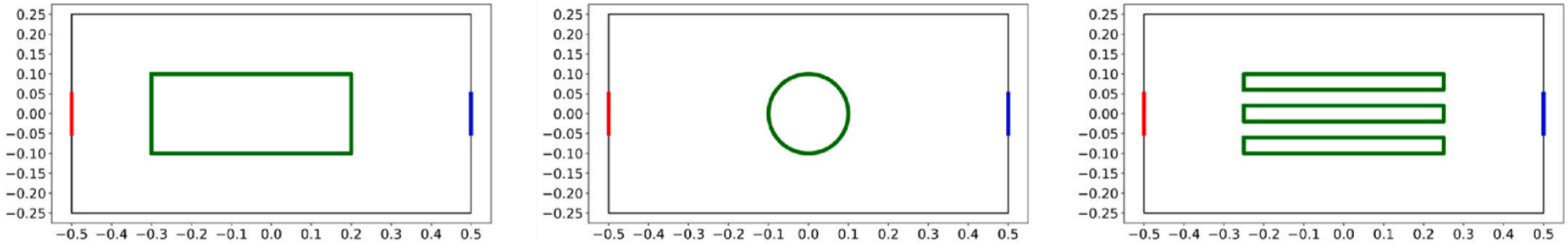




- MFN
- 6 layers with 256 width
- Adaptive GELU
- Evaluated on
  - single box
  - cylinder
  - three-boxes

Collocation Points

- Impact of different number of collocation points



Area	High	Mid	Low
Inlet	500	250	100
Outlet	500	250	100
Channel walls	2500	1250	500
Piece walls	1250	600	250
Channel interior	5000	2500	1000
Total	9750	4850	1950

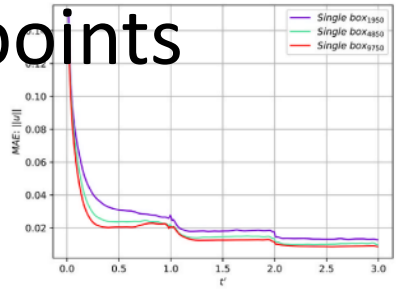
- MFN
- 6 layers with 256 width
- Adaptive GELU
- Evaluated on
  - single box
  - cylinder
  - three-boxes
- In previous tests the Mid set was used

Collocation Points

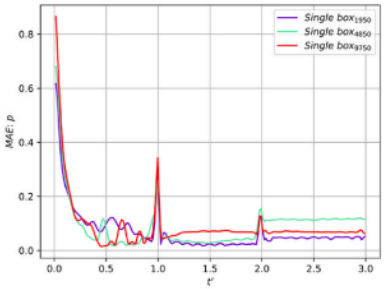
- Impact of different number of collocation points

Geometry	Collocation points	MAE_{avg}			
		$\|\mathbf{u}\|$	u_x	u_y	p
Single box	Low	0.024	0.023	0.013	0.071
	Mid	0.019	0.019	0.010	0.091
	High	**0.017**	**0.016**	**0.009**	0.089
Circular cylinder	Low	0.012	0.012	0.007	0.128
	Mid	0.011	**0.010**	**0.006**	**0.065**
	High	**0.010**	**0.010**	**0.006**	0.219
Three boxes	Low	0.028	0.026	0.016	0.059
	Mid	0.024	0.023	0.014	0.048
	High	**0.021**	**0.019**	**0.012**	**0.112**

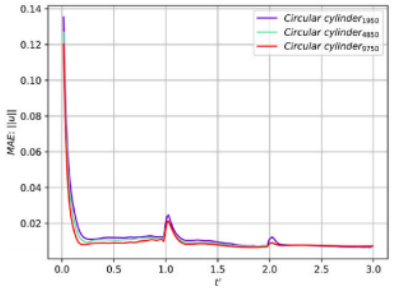
Geometry	Collocation points	MAE_{st}				Training times (hh:mm)
		$\|\mathbf{u}\|$	u_x	u_y	p	
Single box	Low	0.013	0.013	0.006	0.048	**02:47**
	Mid	0.010	0.010	**0.005**	0.116	04:25
	High	**0.009**	**0.008**	**0.005**	**0.068**	07:14
Circular cylinder	Low	**0.007**	**0.006**	**0.004**	0.166	**02:47**
	Mid	**0.007**	**0.006**	**0.004**	**0.040**	04:24
	High	**0.007**	**0.006**	**0.004**	0.226	07:11
Three boxes	Low	0.015	0.014	0.008	0.036	**02:48**
	Mid	0.013	0.012	0.006	**0.021**	04:24
	High	**0.010**	**0.009**	**0.005**	0.126	07:47



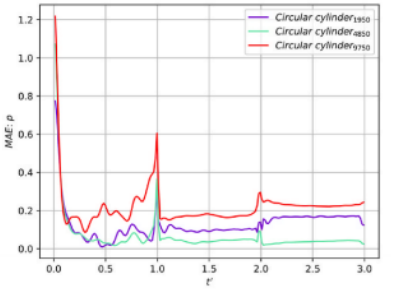
(a)



(b)



(c)



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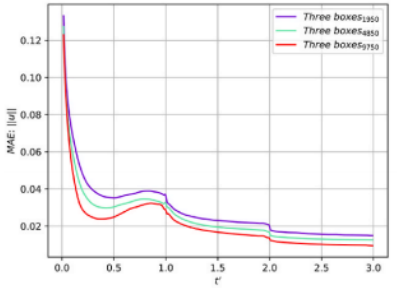
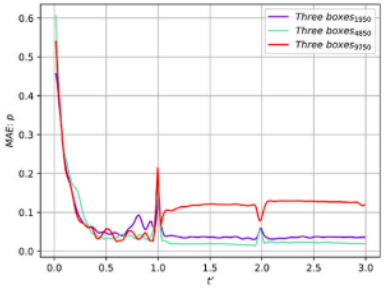


(e)



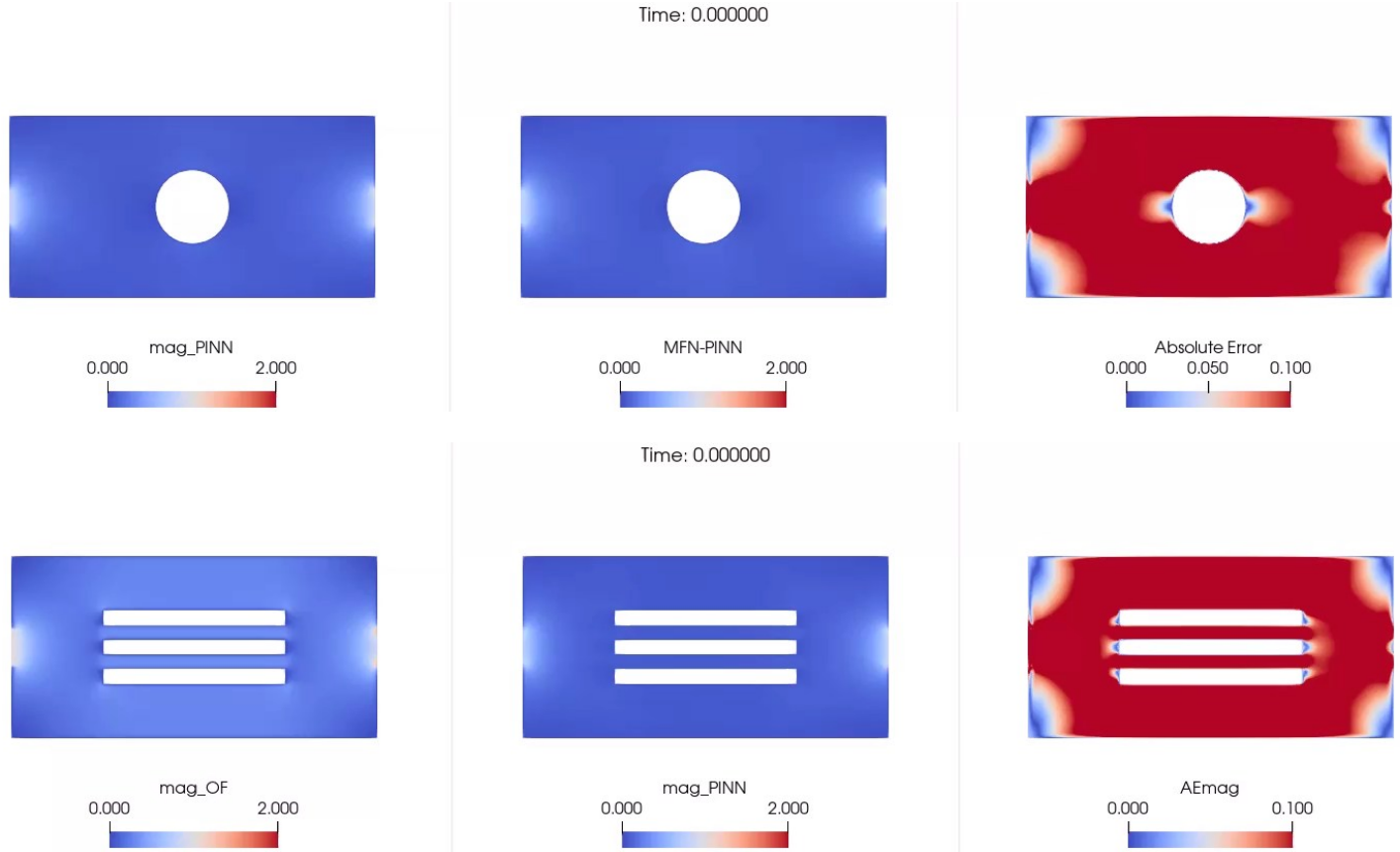
(f)

Advanced Training Strategies

- **Fine-tuning**
  - fine-tune the net trained on the single-box geometry to apply it to the other geometries
  - apply fine-tuning between successive time windows
- **Multi-resolution**
  - number of collocation points changes in different time windows (focus on steady-state solution)
- **Parametrized training**
  - parametrize the piece geometry to learn to predict different (similar) domain without retraining

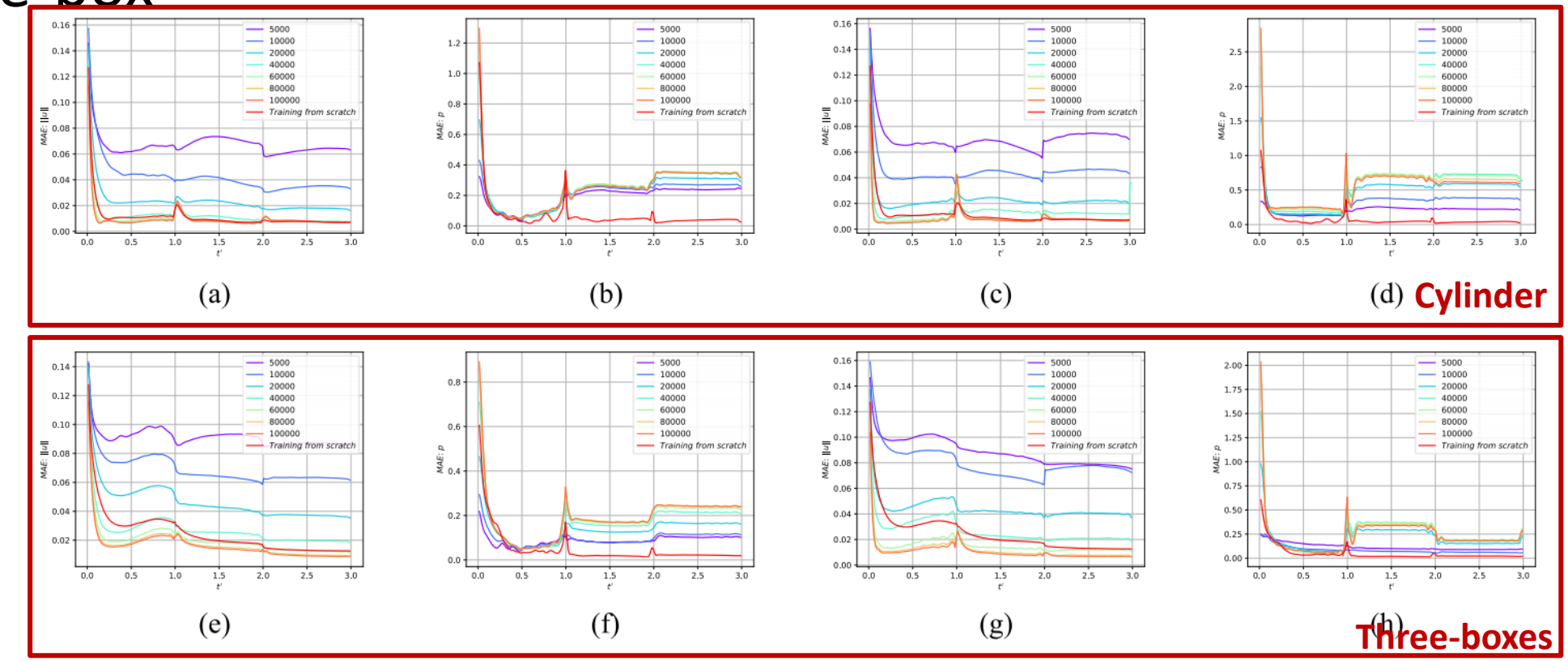
Fine-tuning

- Train on single-box
- Fine-tune on
  - cylinder
  - three-boxes



Fine-tuning

- Train on single-box
- Fine-tune on
  - cylinder
  - three-boxes



$lr=10^{-3}$ $lr=10^{-3}$

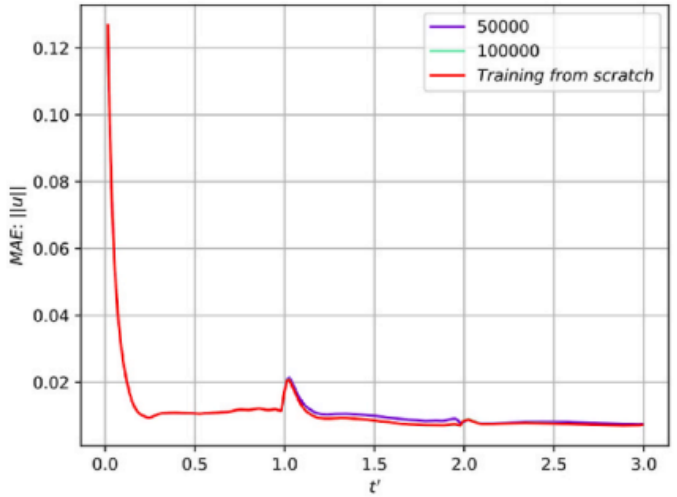
Fine-tuning

Geometry	Training type	Training epochs	MAE_{avg}				MAE_{st}				Training Times (hh:mm)
			$\|\|\mathbf{u}\|\|$	u_x	u_y	p	$\|\|\mathbf{u}\|\|$	u_x	u_y	p	
Single Box to Circular cylinder	Fine-tuning ($lr = 10^{-3}$)	5000	0.067	0.065	0.041	0.188	0.064	0.062	0.039	0.240	**00:13**
		10000	0.042	0.040	0.025	0.209	0.035	0.033	0.022	0.270	00:29
		20000	0.023	0.022	0.014	0.229	0.018	0.017	0.011	0.310	00:59
		40000	0.012	0.012	0.007	0.248	0.008	0.007	0.005	0.342	02:01
		60000	0.010	0.010	0.006	0.252	**0.007**	**0.006**	**0.004**	0.350	03:00
		80000	0.010	0.009	0.005	0.248	**0.007**	**0.006**	**0.004**	0.346	04:01
		100000	0.009	0.009	0.005	0.247	**0.007**	**0.006**	**0.004**	0.343	05:01
	Fine-tuning ($lr = 10^{-2}$)	5000	0.070	0.067	0.037	0.205	0.074	0.071	0.039	0.222	**00:13**
		10000	0.045	0.042	0.025	0.313	0.046	0.044	0.027	0.380	00:29
		20000	0.023	0.021	0.014	0.457	0.022	0.021	0.013	0.585	00:59
		40000	0.014	0.013	0.008	0.561	0.012	0.011	0.007	0.722	02:01
		60000	0.009	0.009	0.005	0.568	**0.007**	**0.006**	**0.004**	0.705	03:01
		80000	**0.008**	**0.008**	**0.004**	0.548	**0.007**	**0.006**	**0.004**	0.651	04:01
		100000	**0.008**	**0.008**	**0.004**	0.531	**0.007**	**0.006**	**0.004**	0.609	05:02
	Training from scratch	100000	0.011	0.010	0.006	**0.065**	**0.007**	**0.006**	**0.004**	**0.040**	04:24
Single Box to Three boxes	Fine-tuning ($lr = 10^{-3}$)	5000	0.091	0.083	0.042	0.088	0.087	0.078	0.04	0.103	**00:13**
		10000	0.069	0.064	0.033	0.096	0.063	0.058	0.026	0.116	00:29
		20000	0.046	0.043	0.023	0.135	0.037	0.034	0.017	0.165	00:59
		40000	0.027	0.025	0.015	0.168	0.019	0.018	0.009	0.214	02:01
		60000	0.020	0.018	0.011	0.181	0.012	0.012	0.006	0.233	03:01
		80000	0.017	0.016	0.010	0.186	0.010	0.009	0.005	0.24	04:01
		100000	0.016	0.015	0.010	0.187	0.009	0.008	0.005	0.242	05:02
	Fine-tuning ($lr = 10^{-2}$)	5000	0.089	0.083	0.038	0.117	0.078	0.072	0.033	0.091	**00:13**
		10000	0.080	0.074	0.036	0.084	0.076	0.069	0.034	0.059	00:29
		20000	0.044	0.041	0.022	0.202	0.040	0.038	0.017	0.157	00:59
		40000	0.027	0.025	0.014	0.235	0.021	0.02	0.009	0.182	02:01
		60000	0.016	0.015	0.009	0.244	0.012	0.011	0.005	0.197	03:01
		80000	0.012	**0.011**	0.007	0.236	0.008	0.007	**0.004**	0.192	04:01
		100000	**0.011**	**0.011**	**0.006**	0.231	**0.007**	**0.006**	**0.004**	0.190	05:02
	Training from scratch	100000	0.024	0.023	0.014	**0.048**	0.013	0.012	0.006	**0.021**	04:24

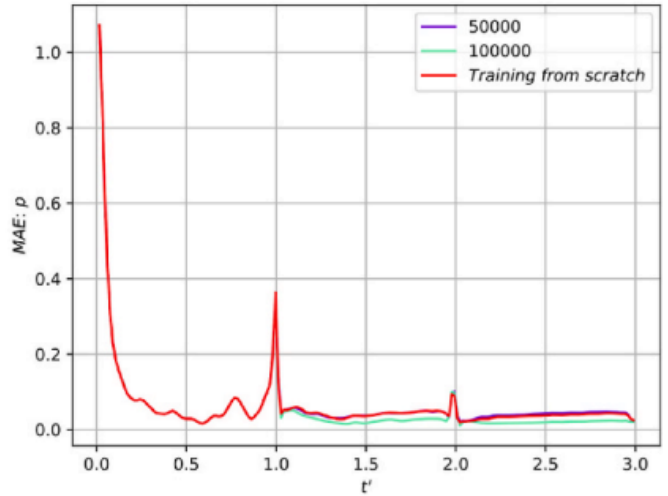
Fine-tuning

- Fine-tuning between time-windows (single-box)

Training epochs	MAE_{avg}				MAE_{st}				Training Times for 2nd and 3rd windows (hh:mm)
	$\|\|\mathbf{u}\|\|$	u_x	u_y	p	$\|\|\mathbf{u}\|\|$	u_x	u_y	p	
50,000	0.012	0.011	0.007	0.066	0.008	0.007	0.005	0.044	**01:38**
100,000	**0.011**	0.011	**0.006**	**0.055**	**0.007**	**0.006**	**0.004**	0.022	03:17
Training from scratch	**0.011**	**0.010**	**0.006**	0.065	**0.007**	**0.006**	**0.004**	**0.040**	03:17



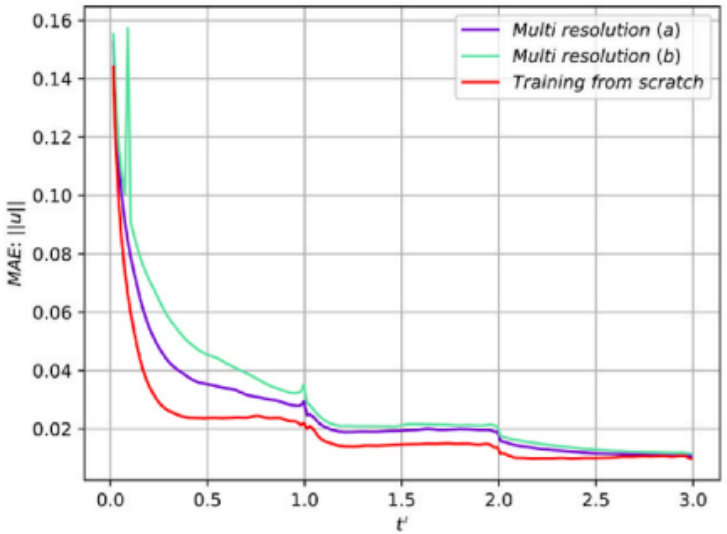
(a)



(b)

Multi-resolution

- Using a reduced number of collocation points in the first two time windows
  - case (a): first and second windows use the Low point configuration (1950)
  - case (b): first and second windows use only 1000 points (Inlet:75, Outlet: 75, Channel Walls: 250, Piece Walls: 100, Channel Interior: 500)
  - in both cases in the last window the High setup is used



(a)

Experiment	MAE_{avg}				MAE_{st}				Training Times (hh:mm)
	$\|\mathbf{u}\|$	u_x	u_y	p	$\|\mathbf{u}\|$	u_x	u_y	p	
Multi resolution (a)	0.026	0.024	0.014	0.057	0.011	0.011	0.006	0.026	01:57 + 02:14
Multi resolution (b)	0.030	0.029	0.015	0.110	0.012	0.012	0.006	**0.101**	**01:36 + 02:14**
Training from scratch	**0.019**	**0.019**	**0.010**	**0.091**	**0.010**	**0.010**	**0.005**	0.116	04:25

Last row uses the Mid setup

Time pass from **04h:25m** to **03h:50m** in case (b)

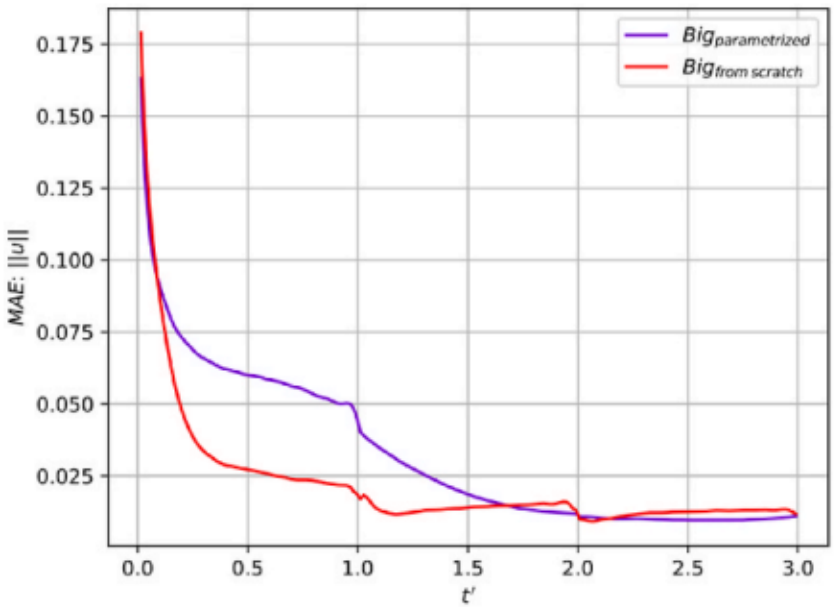
Parametrized learning

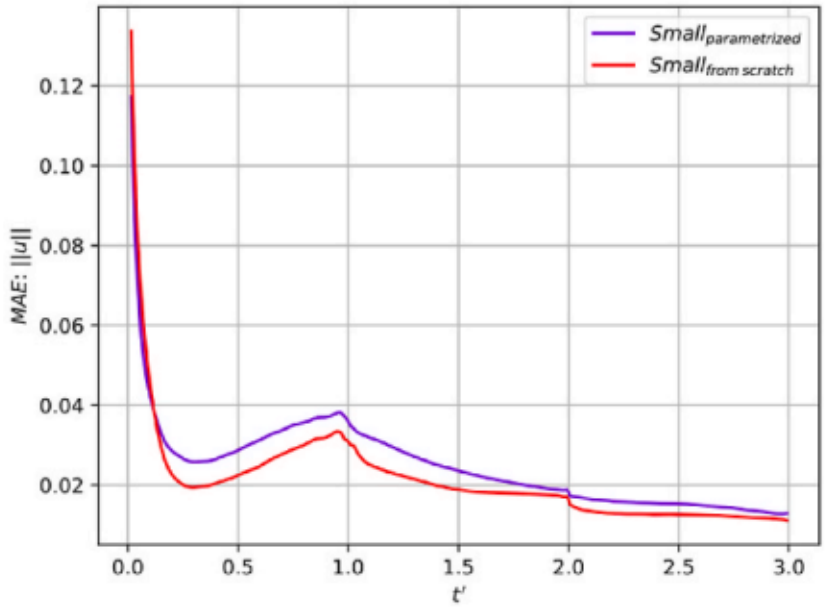
- Box geometry parametrized with
  - width [0.4, 0.6]
  - height [0.15, 0.25]
  - High setup + 150000 epoch (18h:23m)
- Tests considering
  - Big with w=0.6 and h=0.25
  - Small with w=0.4 and h=0.15

Geometry	Training type	MAE_{avg}				MAE_{st}			
		$\|\mathbf{u}\|$	u_x	u_y	p	$\|\mathbf{u}\|$	u_x	u_y	p
Big rectangle	Parameterized	0.032	0.030	0.015	**0.157**	**0.010**	**0.009**	**0.005**	0.163
	Training from scratch	**0.022**	**0.021**	**0.010**	0.192	0.013	0.012	**0.005**	**0.137**
Small rectangle	Parameterized	0.025	0.023	0.017	**0.165**	0.014	0.013	0.009	**0.172**
	Training from scratch	**0.021**	**0.020**	**0.014**	0.331	**0.012**	**0.011**	**0.007**	0.474

Last row uses the Mid setup + 100000 epochs (04h:24m)

Parametrized learning





Geometry	Training type	MAE_{avg}				MAE_{st}			
		$\|\mathbf{u}\|$	u_x	u_y	p	$\|\mathbf{u}\|$	u_x	u_y	p
Big rectangle	Parameterized	0.032	0.030	0.015	**0.157**	**0.010**	**0.009**	**0.005**	0.163
	Training from scratch	**0.022**	**0.021**	**0.010**	0.192	0.013	0.012	**0.005**	**0.137**
Small rectangle	Parameterized	0.025	0.023	0.017	**0.165**	0.014	0.013	0.009	**0.172**
	Training from scratch	**0.021**	**0.020**	**0.014**	0.331	**0.012**	**0.011**	**0.007**	0.474

Last row uses the Mid setup + 100000 epochs (04h:24m)